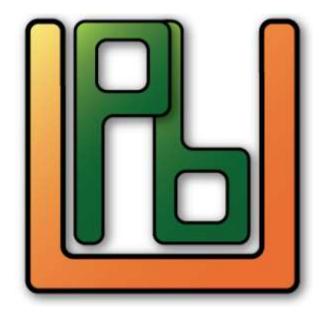
U-Pb_Redux

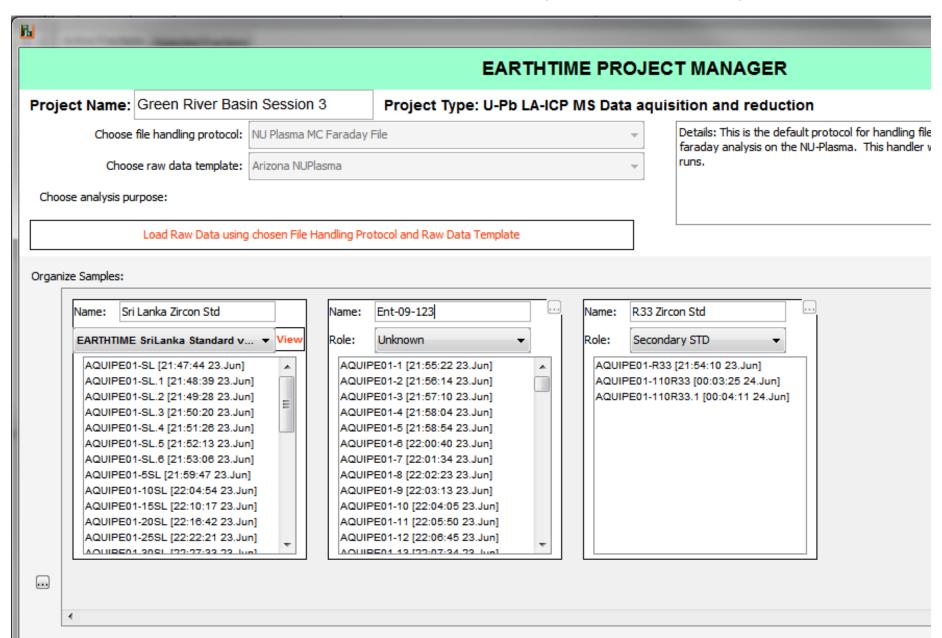


Noah McLean, Jim Bowring

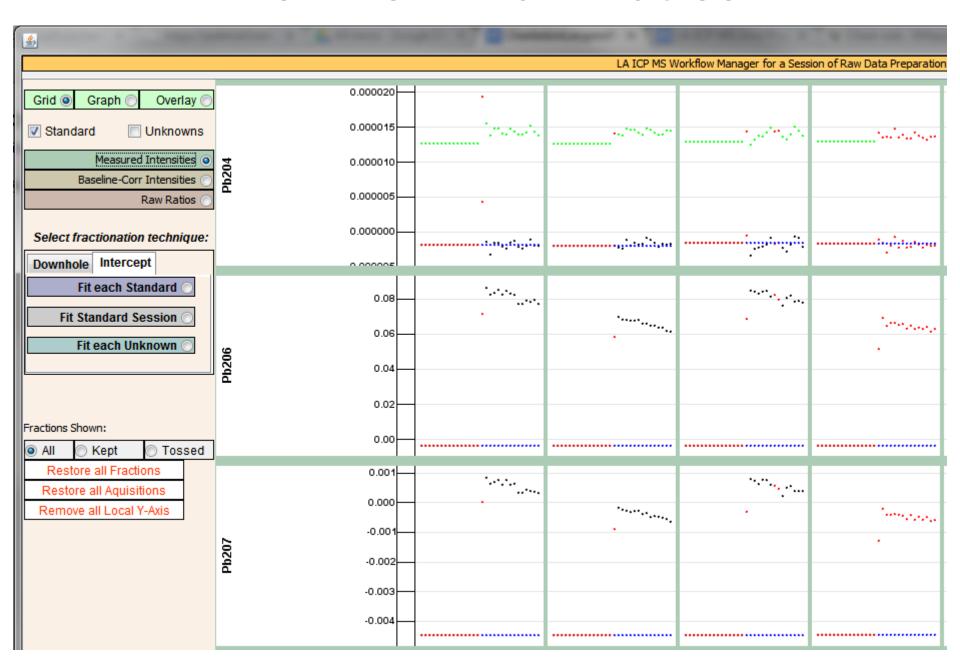
Outline:

- 1. Uncertainty propagation protocol/workflow
- 2. Common Pb correction methods, if applicable
- 3. Method of inter-element and inter-isotope fractionation correction
- 4. Weighted Mean/Linear regression support, if applicable
- 5. Rejection criteria
- Handling/storage of reference values for normalization
- 7. Key differences from other available packages

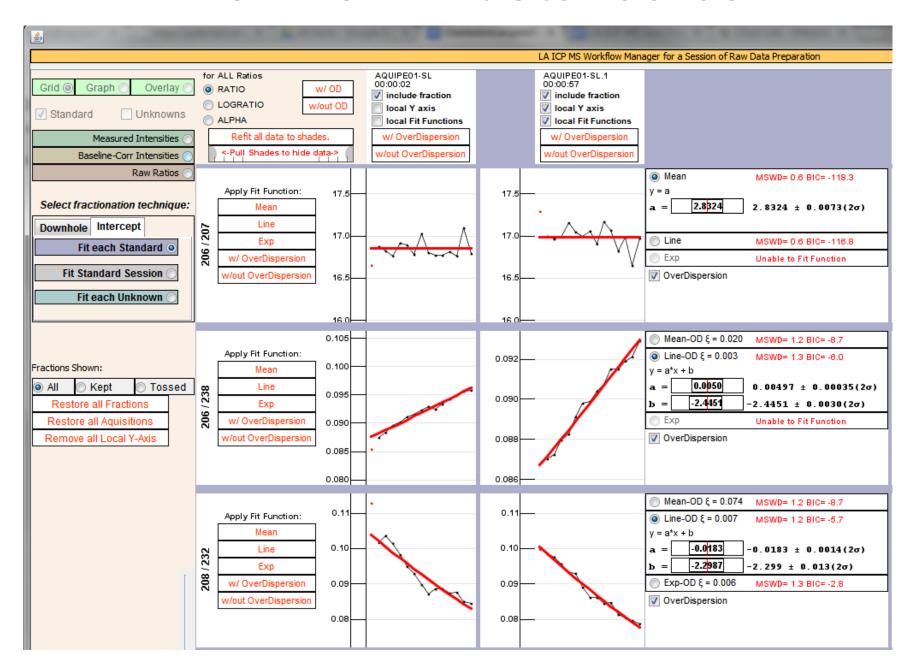
Workflow: Sample Setup



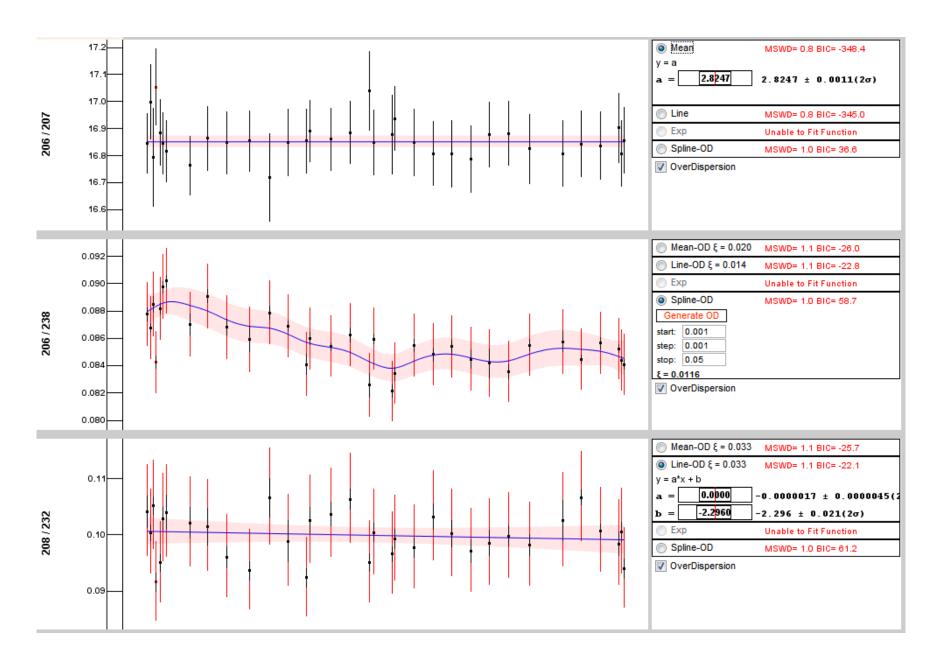
Workflow: Raw Ratios



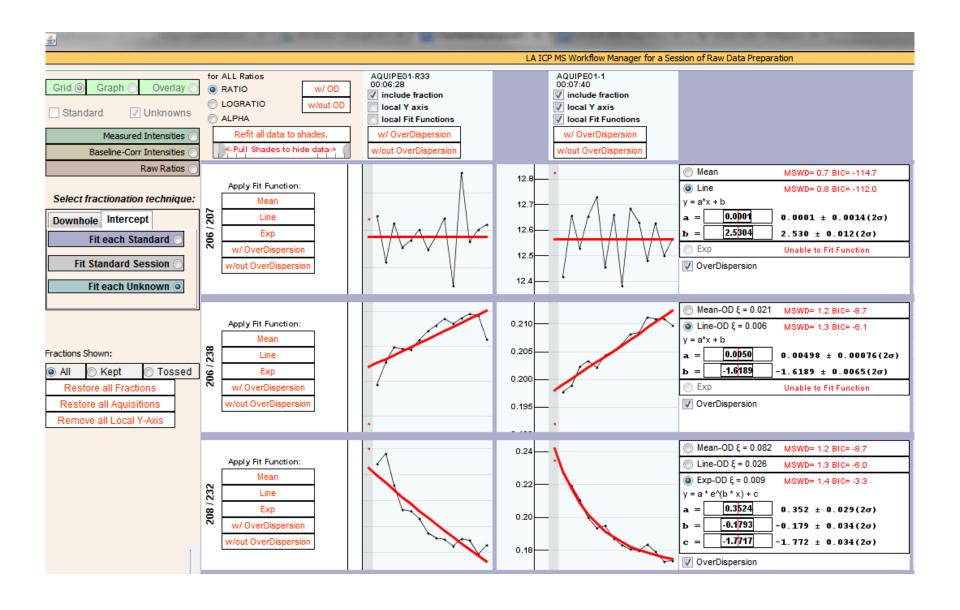
Workflow: Fit Standards



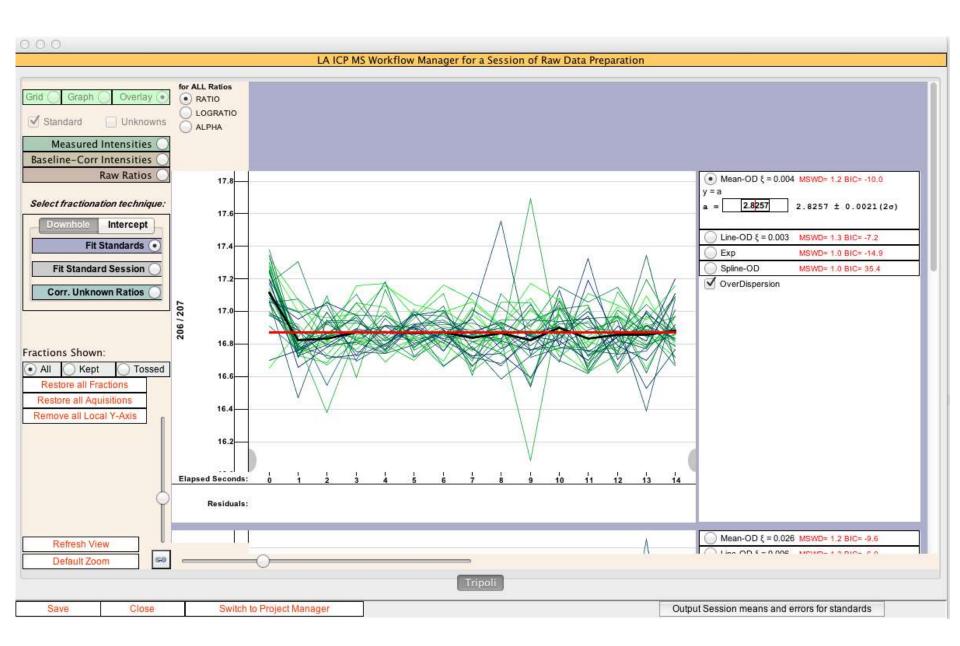
Workflow: Fit Session



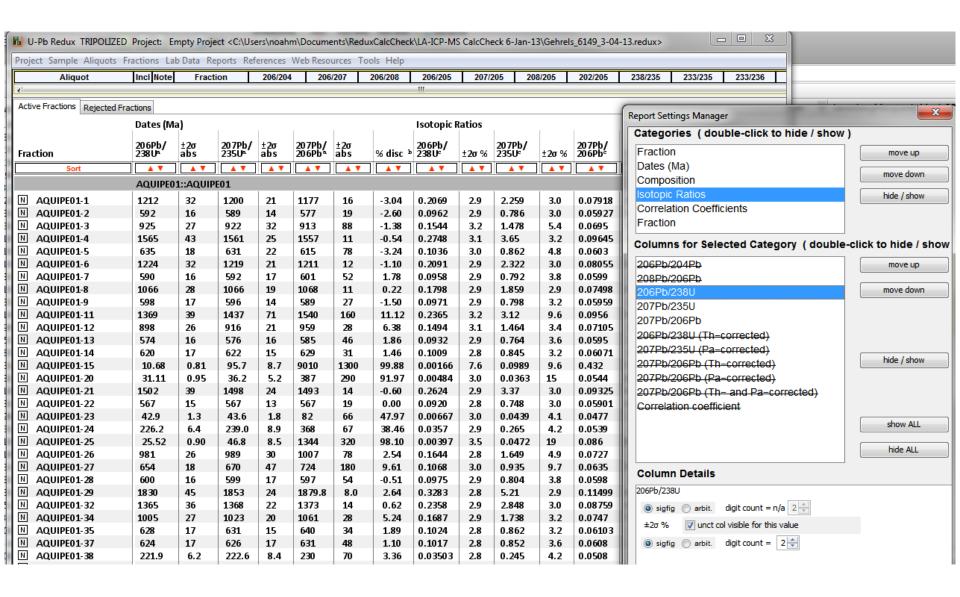
Workflow: Fit Unknowns



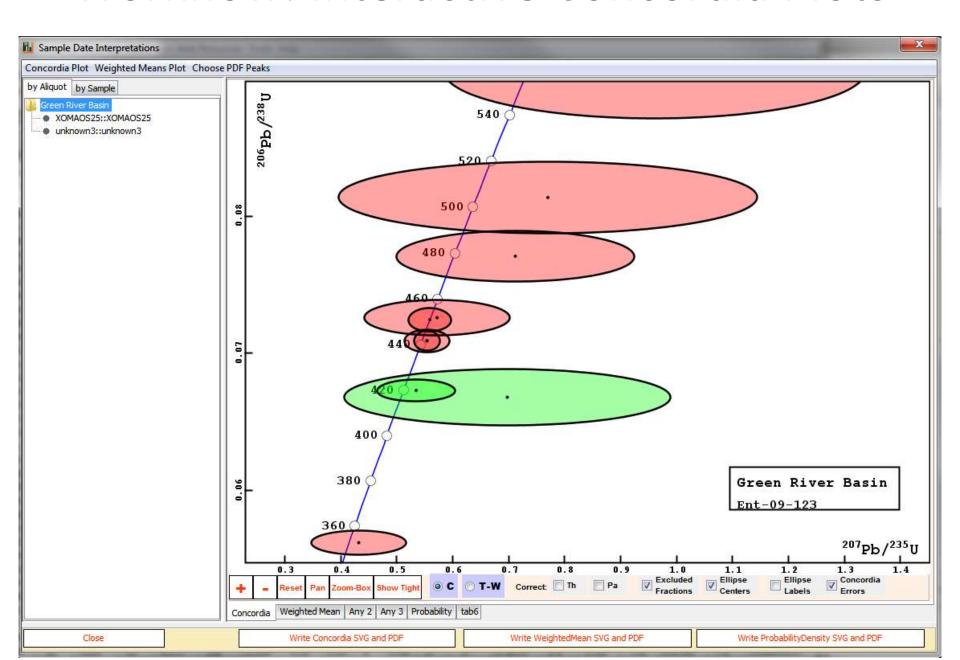
Workflow: Downhole (Iolite)



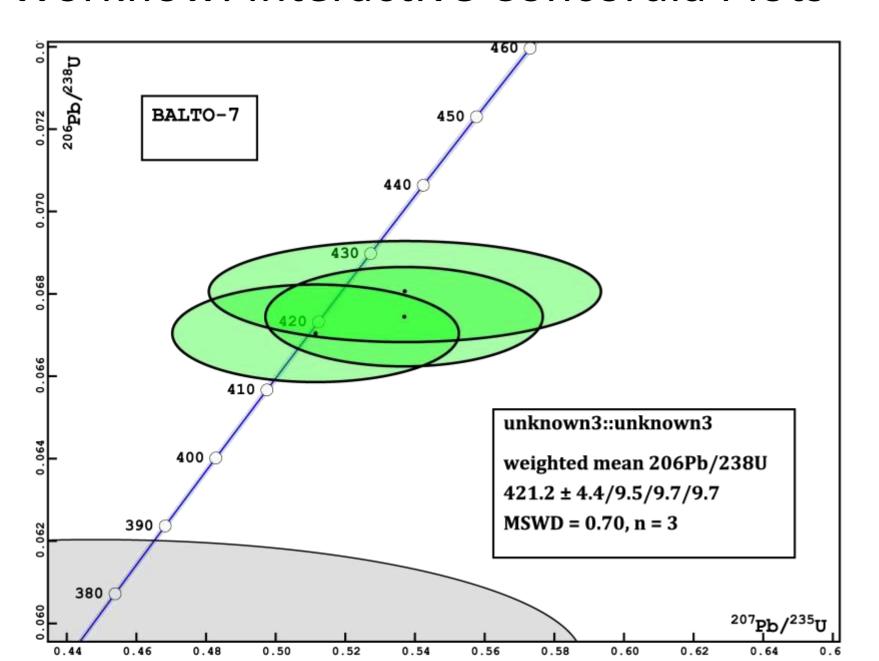
Workflow: Interactive Data Table



Workflow: Interactive Concordia Plots



Workflow: Interactive Concordia Plots



2. Common Pb corrections

 Isochron regression methods available (upper/lower intercept from TIMS)

3. Fractionation Correction

$$\left(\frac{x}{y}\right)_{true} = A\left(\frac{x}{y}\right)_{meas}$$

$$\log\left(\frac{x}{y}\right)_{true} = \alpha + \log\left(\frac{x}{y}\right)_{meas}$$

4. Weighted Mean/Linear Regression

- Weighted means take into account reported in the form X/Y/Z/W, where
 - X is the analytical uncertainty, and takes into account the uncertainty correlations from samplestandard bracketing
 - Y also includes the lab's observed variability in multiple standards of the same mineral
 - Z additionally includes uncertainties in the mineral standard IC
 - W adds in decay constant uncertainties

5. Rejection Criteria

- None: user-based rejection.
 - Open to suggestions, but 2σ ends up underestimating uncertainties

6. Handling/Storage of Reference Values

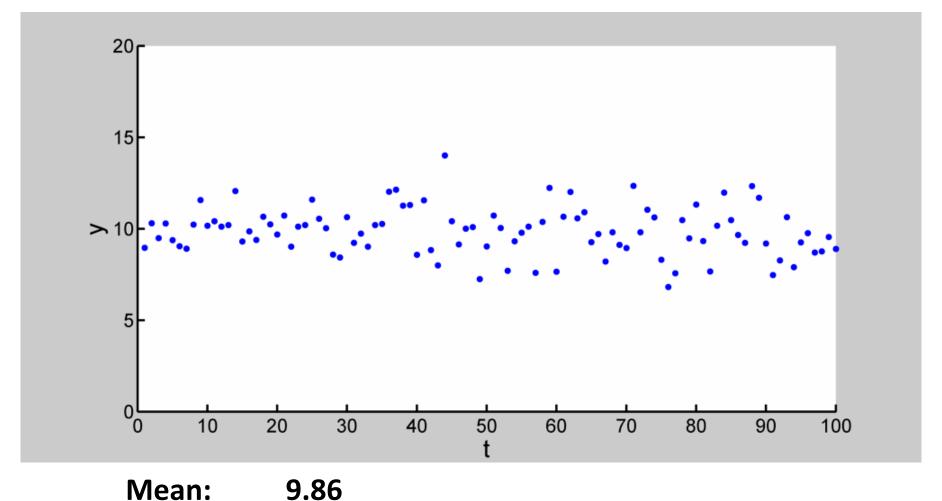
- Model stored publicly online as xml
 - Separately records radiogenic end-member and common Pb ICs, with covariance matrix to describe the uncertainties in each

7. Key Differences

- A. Treats heteroscedastic and overdispersed data
- B. Log-ratio analysis of compositional data
- C. Propagates systematic uncertainties between integrations dead time, detector grains/non-linearity
- D. Provides model selection guidance (BIC)
- E. Treats systematic uncertainties between measurements from sample-standard bracketing
- F. Plugs into ID-TIMS Redux for plotting, weighted means, databasing capabilities

A. Heteroscedastic data

Uncertainty in each data point is different

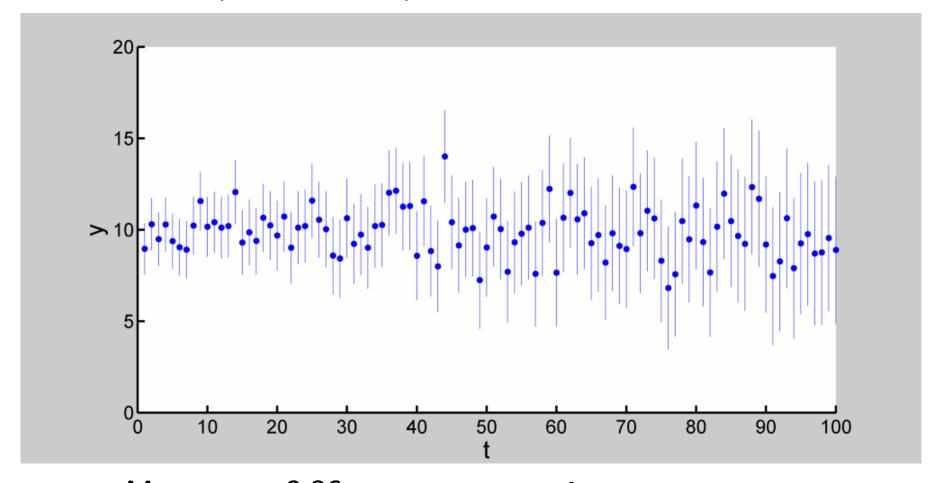


Mean:

0.26 ±2SE:

A. Heteroscedastic data

Uncertainty in each data point is different



Mean:

9.86

Wtd Mean:

9.94

±2SE:

0.26

±2σ

0.24

A. Heteroscedastic data

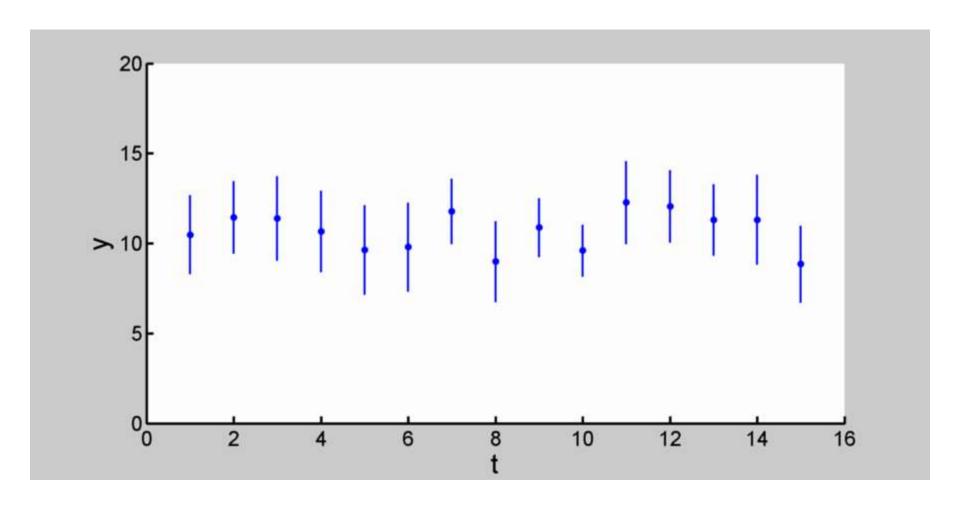
- Estimate uncertainty in each analysis
 - Total counts (shot noise)
 - Amplifier behavior (Johnson noise)
- If the measured variability in the data is larger than the estimated uncertainties, make up the difference with overdispersion (excess variance)

B. Log-ratios of compositional data

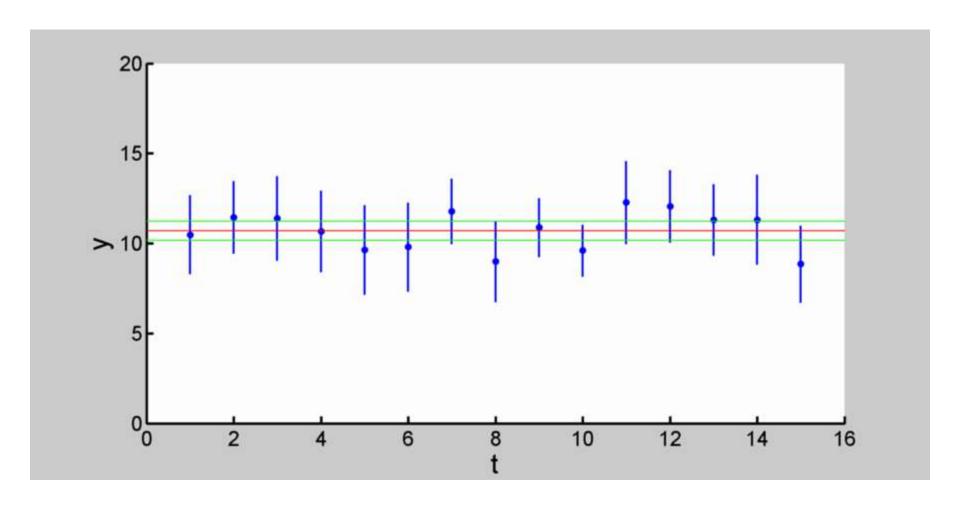
- Resolves the age-old problem: ratio of the mean vs. mean of the ratios?
- Differences between isotope ratios do not obey the rules of 'distances': evaluating means and standard deviations generates internally inconsistent results.
- Instead, use e.g. log(207/206), log(208/206)
- Evaluate math in log-ratio space, then exponentiate. More Thursday!

C. Propagate systematic uncertainties between successive integrations

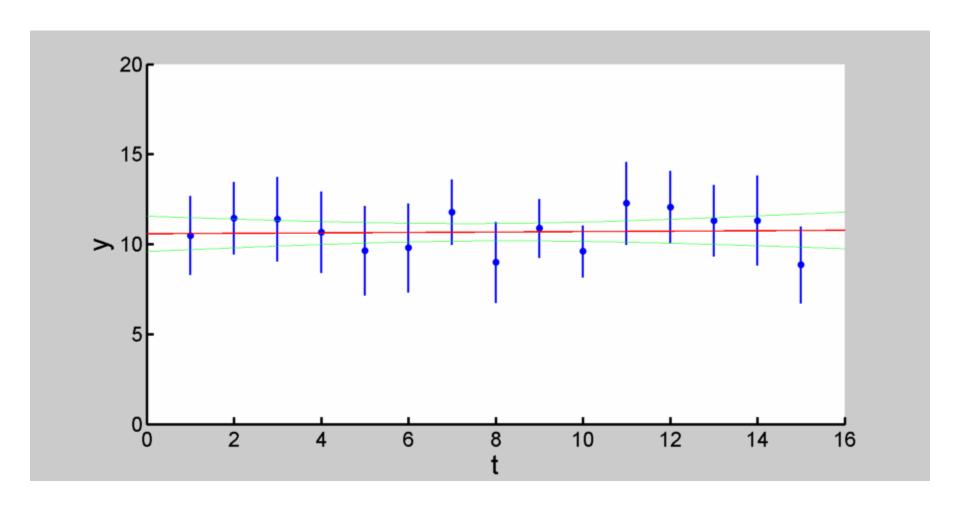
- Not as important for LA-ICP-MS at present, but will become more so as uncertainties continue to come down
- **Dead time**, ion counter non-linearity, gain inter-calibration between detectors.



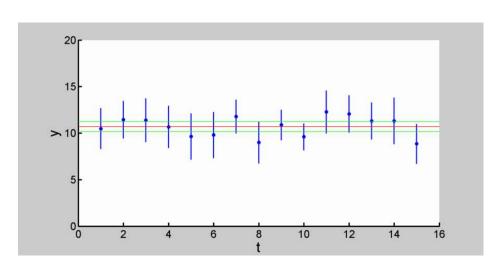
Do these measured data have a trend? Should I use a mean or a linear fit to describe the data?



Weighted mean: $10.69 \pm 0.53 (2\sigma)$, MSWD = 1.38

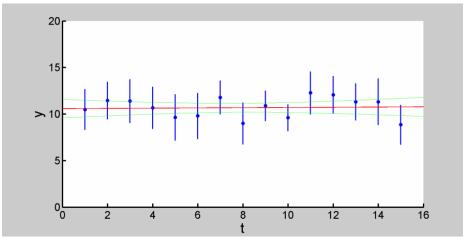


Line fit: $(0.012 \pm 0.11)t + (10.58 \pm 0.98)$, MSWD = 1.13 $\rho_{ab} = -0.871$



Weighted mean: $10.69 \pm 0.53 (2\sigma)$,

MSWD = 1.38, **BIC = 20.7**

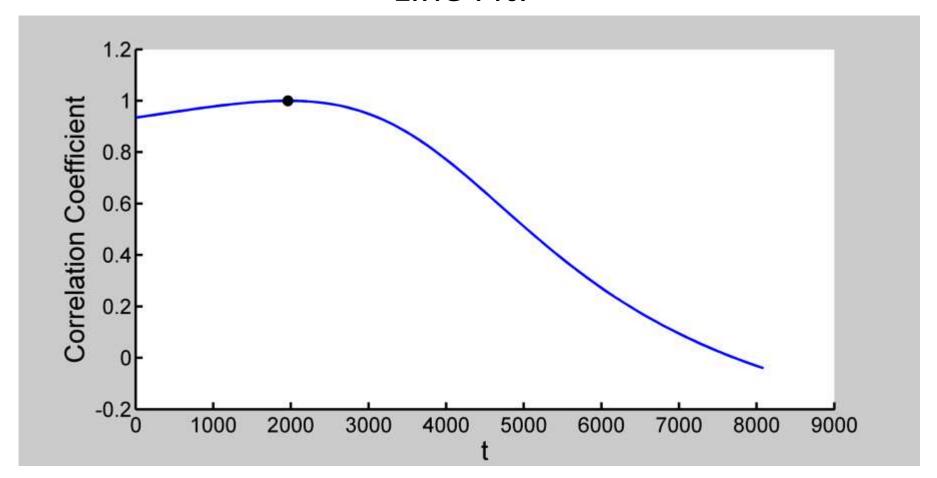


Line fit: $(0.012 \pm 0.11)t + (10.58 \pm 0.98)$, MSWD = 1.13, **BIC = 19.9**

Lowest BIC wins: the line is most likely the best fit.

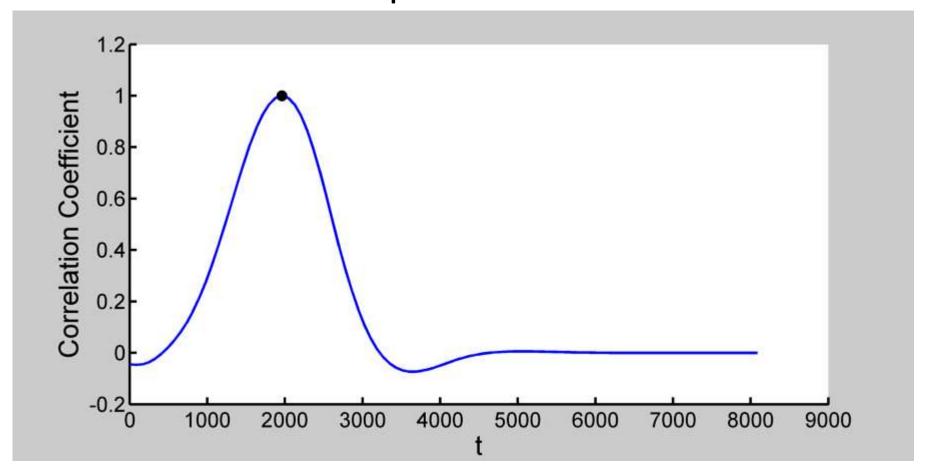
E. Sample-standard bracketing → Unct. correlation between unknowns

Line Fit:

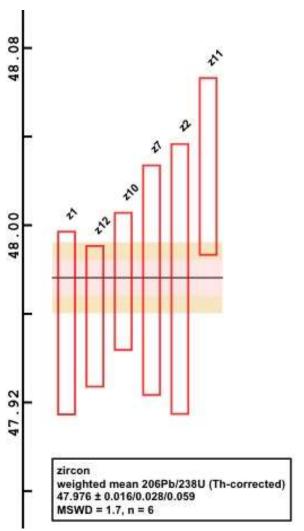


E. Sample-standard bracketing → Unct. correlation between unknowns

Spline Fit:



F. Calculating a weighted mean



 Gives more weight to more precise analyses, un-weights less precise analyses

$$\bar{t} = \sum_{i=1}^{n} \alpha_i t_i = \sum_{i=1}^{n} \left(\frac{t_i}{\sigma_i^2}\right) / \sum_{i=1}^{n} \left(\frac{1}{\sigma_i^2}\right)$$

But there is no room here for systematic uncertainties, from sample-standard bracketing, standard ICs, decay constants...

Systematic uncertainties are covariance

$$\bar{t} = \alpha_1 t_1 + \alpha_2 t_2 \qquad \alpha_1 + \alpha_2 = 1$$

$$\sigma_{\bar{t}}^2 = \begin{bmatrix} \frac{d\bar{t}}{dt_1} & \frac{d\bar{t}}{dt_2} \end{bmatrix} \begin{bmatrix} \sigma_{t_1}^2 & \sigma_{t_1 t_2}^2 \\ \sigma_{t_1 t_2}^2 & \sigma_{t_2}^2 \end{bmatrix} \begin{bmatrix} \frac{d\bar{t}}{dt_1} \\ \frac{d\bar{t}}{dt_2} \end{bmatrix}$$

What are the weights that minimize the uncertainty in the weighted mean?

Systematic uncertainties are covariance

$$\bar{t} = \alpha_1 t_1 + \alpha_2 t_2 \qquad \alpha_1 + \alpha_2 = 1$$

$$\sigma_{\bar{t}}^2 = \begin{bmatrix} \frac{d\bar{t}}{dt_1} & \frac{d\bar{t}}{dt_2} \end{bmatrix} \begin{bmatrix} \sigma_{t_1}^2 & \sigma_{t_1 t_2}^2 \\ \sigma_{t_1 t_2}^2 & \sigma_{t_2}^2 \end{bmatrix} \begin{bmatrix} \frac{d\bar{t}}{dt_1} \\ \frac{d\bar{t}}{dt_2} \end{bmatrix}$$

$$\alpha = \mathbf{\Sigma}^{-1} \mathbf{1} / (\mathbf{1}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \mathbf{1})$$

Systematic uncertainties are covariance

$$\bar{t} = \alpha_1 t_1 + \alpha_2 t_2 \qquad \alpha_1 + \alpha_2 = 1$$

$$\sigma_{\bar{t}}^2 = \begin{bmatrix} \frac{d\bar{t}}{dt_1} & \frac{d\bar{t}}{dt_2} \end{bmatrix} \begin{bmatrix} \sigma_{t_1}^2 & \sigma_{t_1 t_2}^2 \\ \sigma_{t_1 t_2}^2 & \sigma_{t_2}^2 \end{bmatrix} \begin{bmatrix} \frac{d\bar{t}}{dt_1} \\ \frac{d\bar{t}}{dt_2} \end{bmatrix}$$

$$\bar{t} = \mathbf{1}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \mathbf{t} / (\mathbf{1}^{\mathrm{T}} \mathbf{\Sigma}^{-1} \mathbf{1})$$



Software Chasm

Kelly: domain-independent software engineering solutions serve to isolate the scientific-computing community

[IEEE Software 2007]

Wilson: treat scientific software with scientific rigor [American Scientist 2006]



Goal:

calibrate earth history and develop the geochronological techniques necessary to produce dates with uncertainties approaching 0.1 percent

Requires:

robust software for data reduction, analysis, and archiving to support both science and education

EARTHTIME Software Requirements

open-source applications for full U-Pb data reduction and error propagation

provide graphical and statistical tools

produce publication-ready artifacts

produce a standardized record for transport to/from archival databases

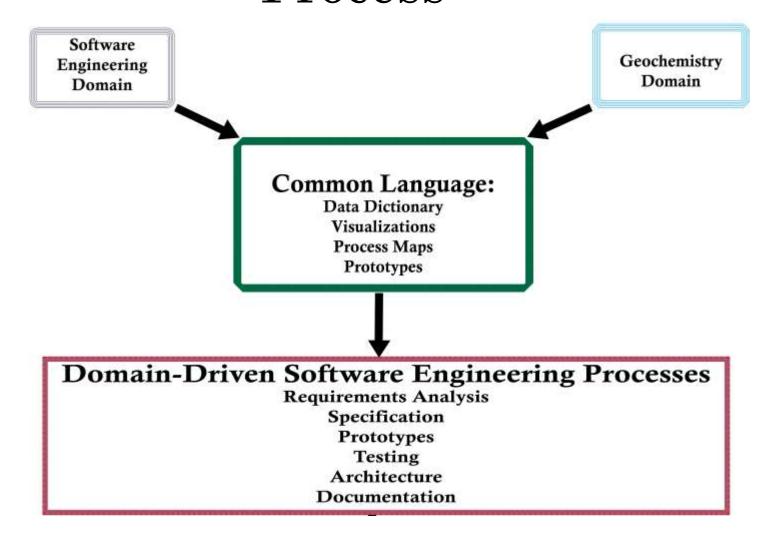
CIRDLES

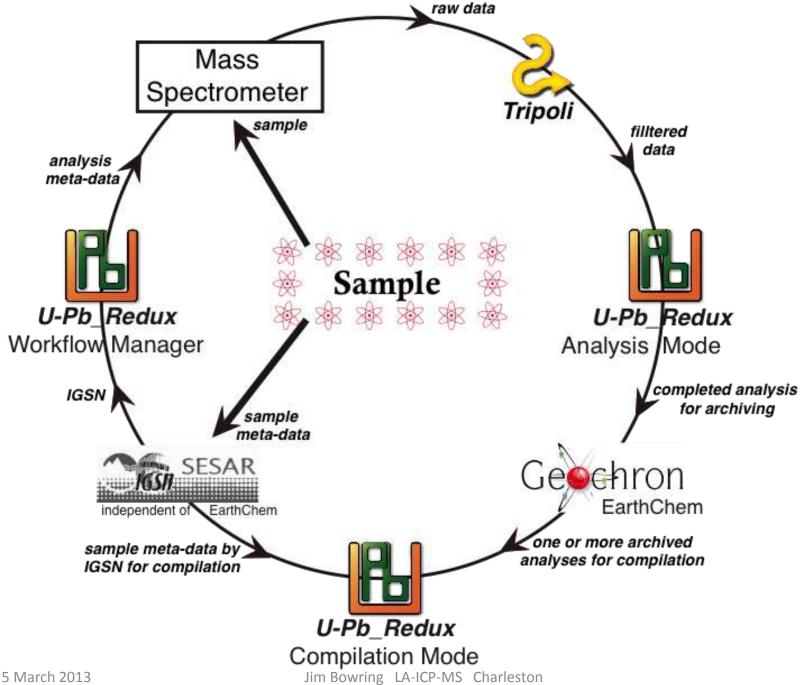
<u>Cyber Infrastructure Research and Development</u> <u>Lab for the Earth Sciences</u>

CIRDLES.org

Collaborative domain-specific software engineering research to produce tools that advance science

Domain-Driven Development Process





Questions?

